**The Multiple Regression Case: Deriving OLS with Matrices**

We will usually be trying to predict a dependent variable using scores from several independent variables. Deriving a more general form of the least-squares estimator for this situations we us matrix operations. The basic OLS regression equation can be represented in the following matrix form:

Y = XB + e (1)

column matrix of cases scores on the DV

matrix of cases on IV [First column is placeholder column of ones for the constant and the remaining columns correspond to each IV].

column matrix containing regression constant and coefficients

column matrix of cases error of prediction.

To find the values for the elements of B which minimize the sum of squared errors. This quantity which we are trying to minimize can be expressed

In order to take derivative of the quantity with regard to the matrix B, we need to first express e in terms of Y, X and B:

Substituting the expression on the right side into equation (1) we get

For any two matrices A and B which can be multiplied together

This product will be computed:

(2)

Middle term known as “linear form” in B which is equivalent of a scalar term in which the variable we are differentiating with respect to is raised to the first power (i.e. a linear term) which means we obtain the derivative by dropping the B and taking transpose of all the matrices in the expression. It gives us

Third term known as quadratic form in B. This means we obtain derivative by dropping from the term and multiplying by two which gives us . The full partial derivative is

(3)

We then subtract from each side of the equation

Finally, we need to solve for B by pre-multiplying each side of the equation the inverse of . Remember that this is the matrix equivalent of dividing each side of the equation by

(4)

This equation (4) is familiar OLS estimator. While gives the sum of cross-products of X and Y, gives us the sum of squares for X. Since pre-multiplying by is the matrix equivalent by dividing by , this expression is basically doing the same thing as scalar expression for : dividing the sum of cross products of the IV (or IVs) and the DV by the sum of squares for the IV (or IVs).